Bred Vectors: A simple tool to understand complex dynamics

With deep gratitude to Akio Arakawa

for all the understanding he has given us

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Arakawa

- I never had the privilege of taking a course from Akio…but
- his NWP class notes were fundamental guidance
- on some areas I did not agree completely
- many of his ideas went way over my head
- but I learned so much from him!
- Akio profoundly influenced my book…
- "a short but inspiring introduction is presented in Arakawa (1997)", "the NWP class notes of Arakawa at UCLA…"
- **Thank you Akio!**

Contents: a simple tool to understand complex dynamics

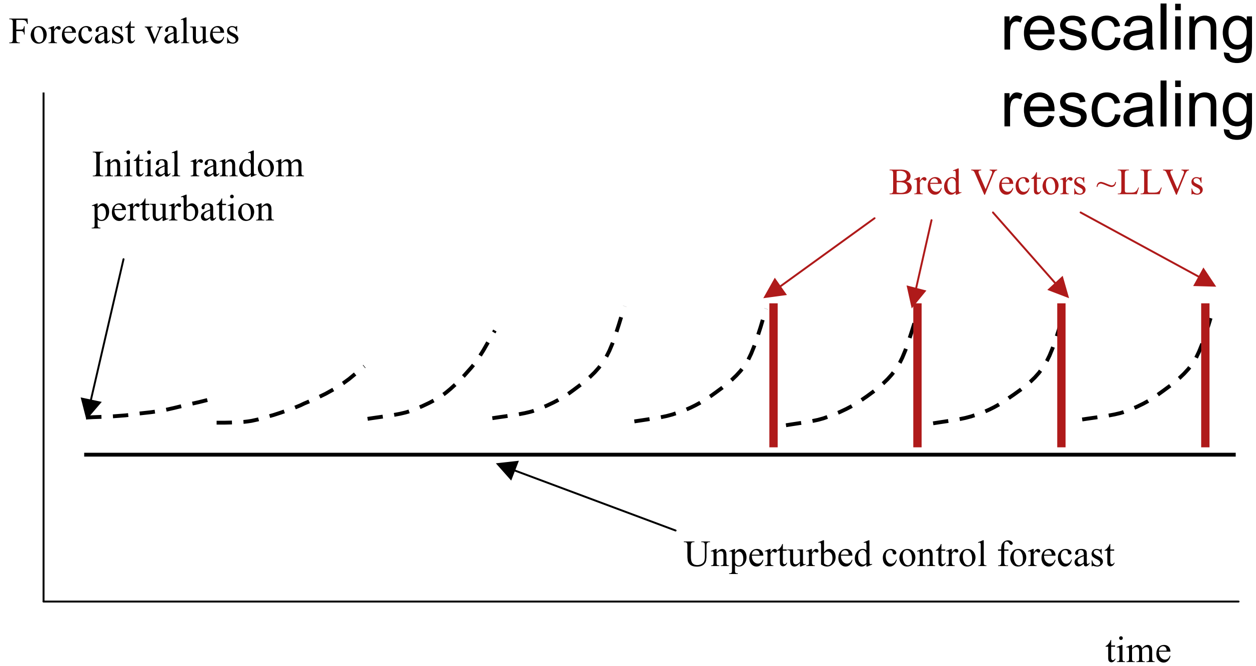
- What is breeding?
  - Running any nonlinear model twice!… just a black box
  - This finds the most unstable normal modes in an evolving flow
- Example: breeding with the Lorenz model
  - Undergraduates interns found that with breeding they could easily predict Lorenz regime changes and their duration
  - Coupled slow-fast model: can get both slow and fast instabilities
- Bred vectors reflect the “errors of the day”
  - Can be used for ensemble prediction and data assimilation
- Example: ENSO bred vectors with the NASA/NCEP CGCMs
  - Predict evolution of errors, Improve seasonal ensemble prediction
- Example: Exploring <u>all</u> the instabilities in the ocean
  - Explaining instabilities with the KE equation for bred vectors

The atmosphere has coupled instabilities that span many time scales:

- ENSO has a doubling time of about one month, baroclinic waves about 2 days, cumulus convection about 10 minutes…
- Linear approaches (like Singular Vectors and Lyapunov Vectors) can only handle the fastest instability.
- Nonlinear model integrations (like Bred Vectors, EnKF) allow fast instabilities to saturate, they can filter out fast instabilities
- This allows Bred Vectors to isolate either fast or slow modes by choosing the amplitude and rescaling **time interval**.

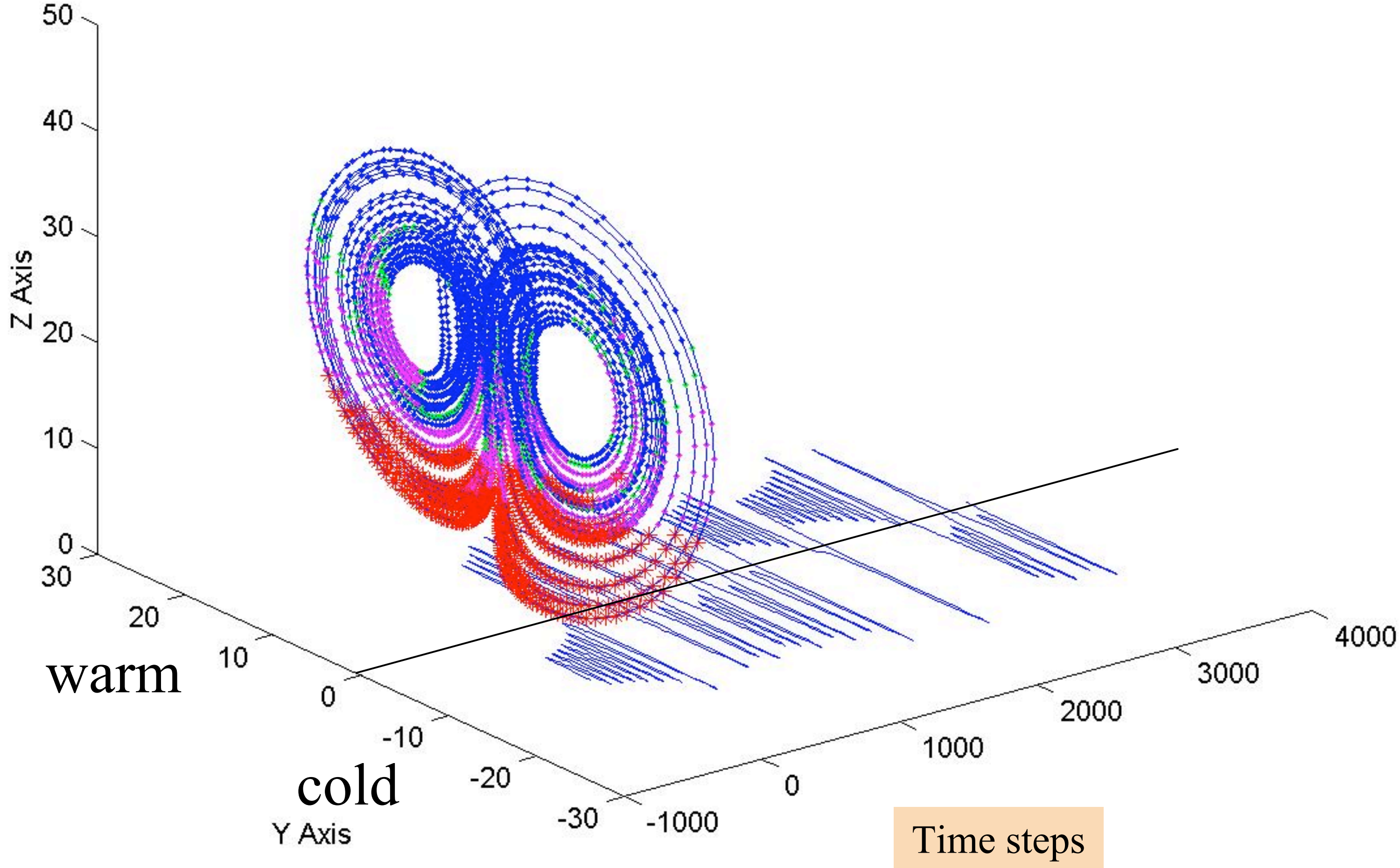
Breeding: simply running the nonlinear model a second time, from perturbed initial conditions.

Only two tuning parameters: rescaling amplitude and rescaling interval

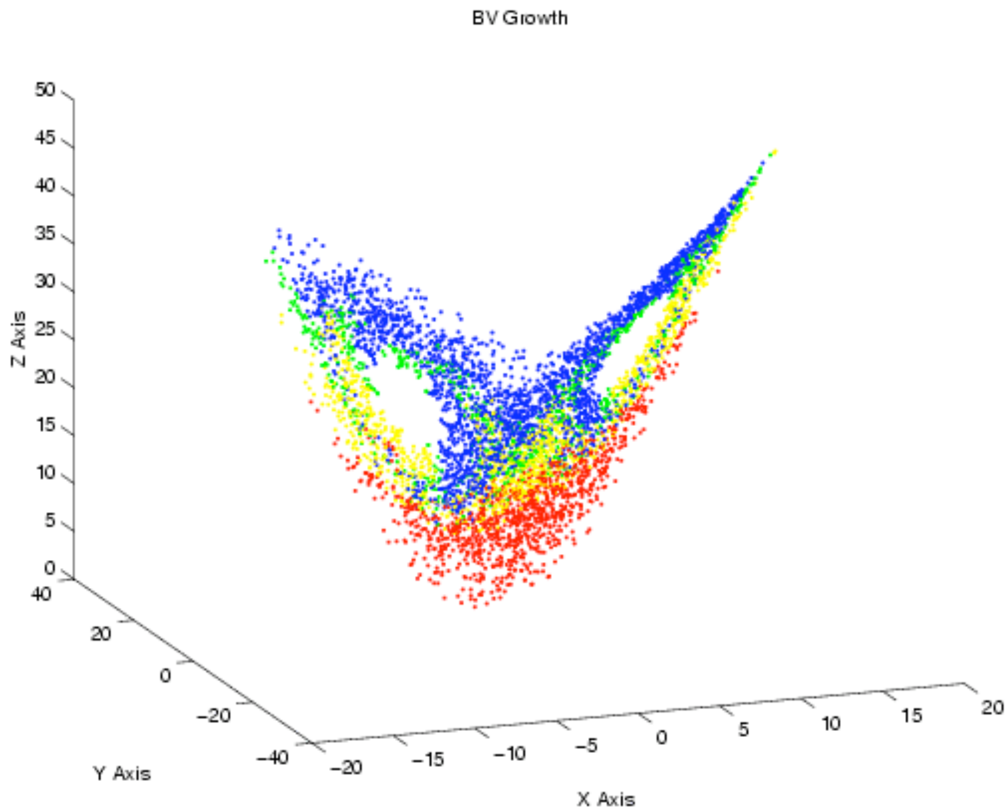


Local breeding growth rate: $$g(t) = \frac{1}{n\Delta t} \ln\left(|\delta \mathbf{x}| / |\delta \mathbf{x}_0| \right)$$

4 summer undergraduates computed the Lorenz BV growth rate: red is large BV growth, blue is decay



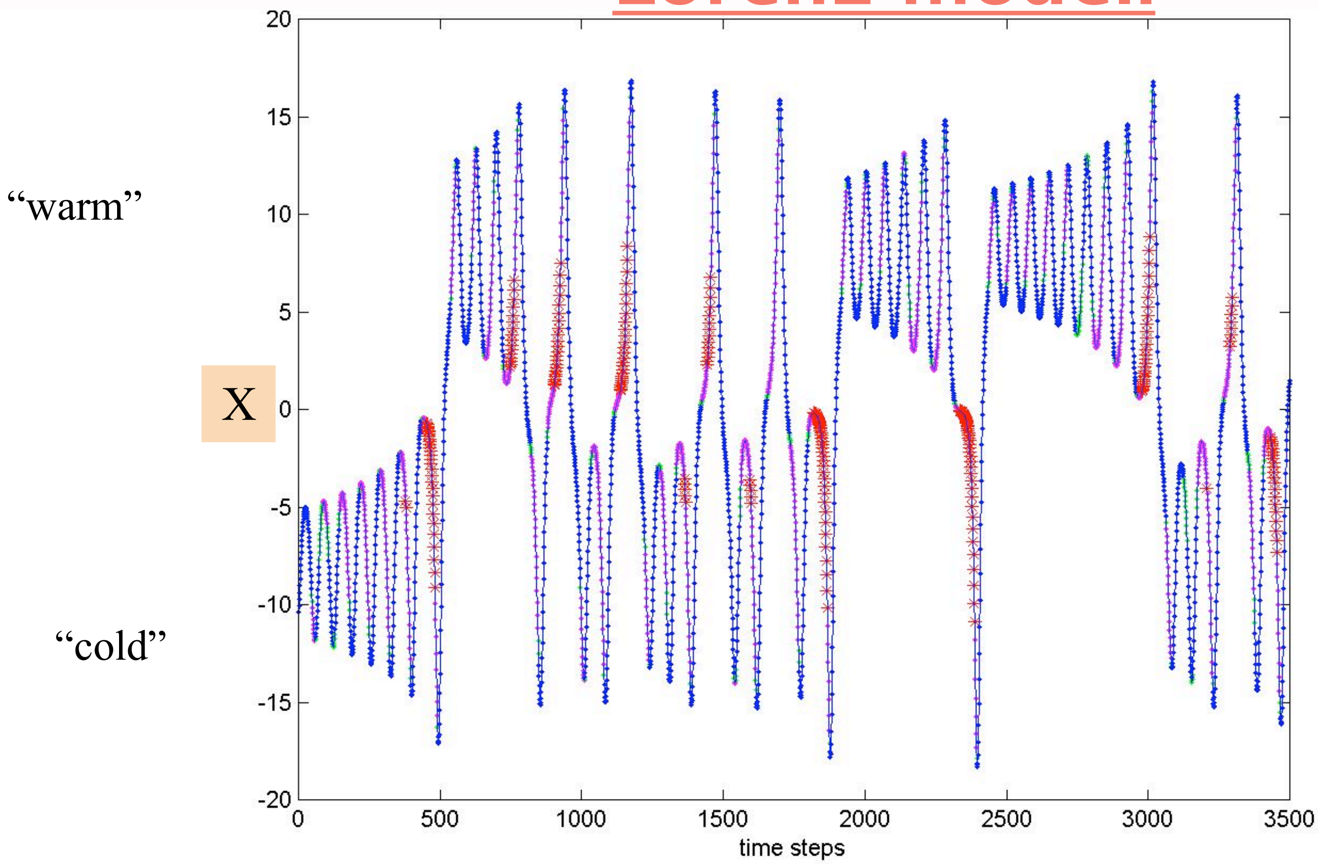
In the 3-variable Lorenz (1963) model we used breeding to estimate the local growth of perturbations:



Bred Vector Growth:
red, high growth;
yellow, medium;
green, low growth;
blue, decay

With just a single breeding cycle, we can estimate the stability of the attractor (Evans et al, 2004)

Discovered forecasting rules for the Lorenz model:



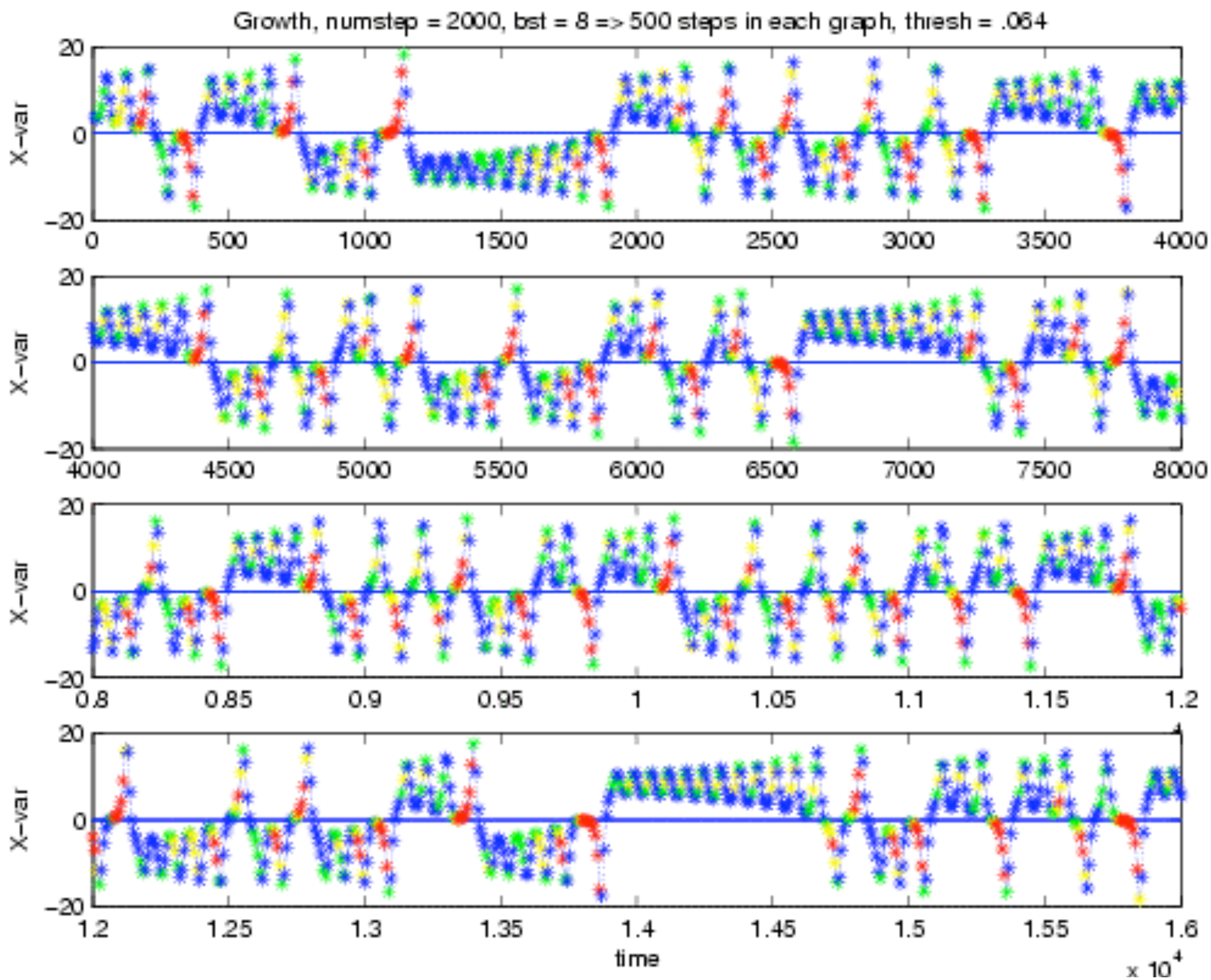
Growth rate of bred vectors:

A * indicates fast growth (>1.8 in 8 steps)

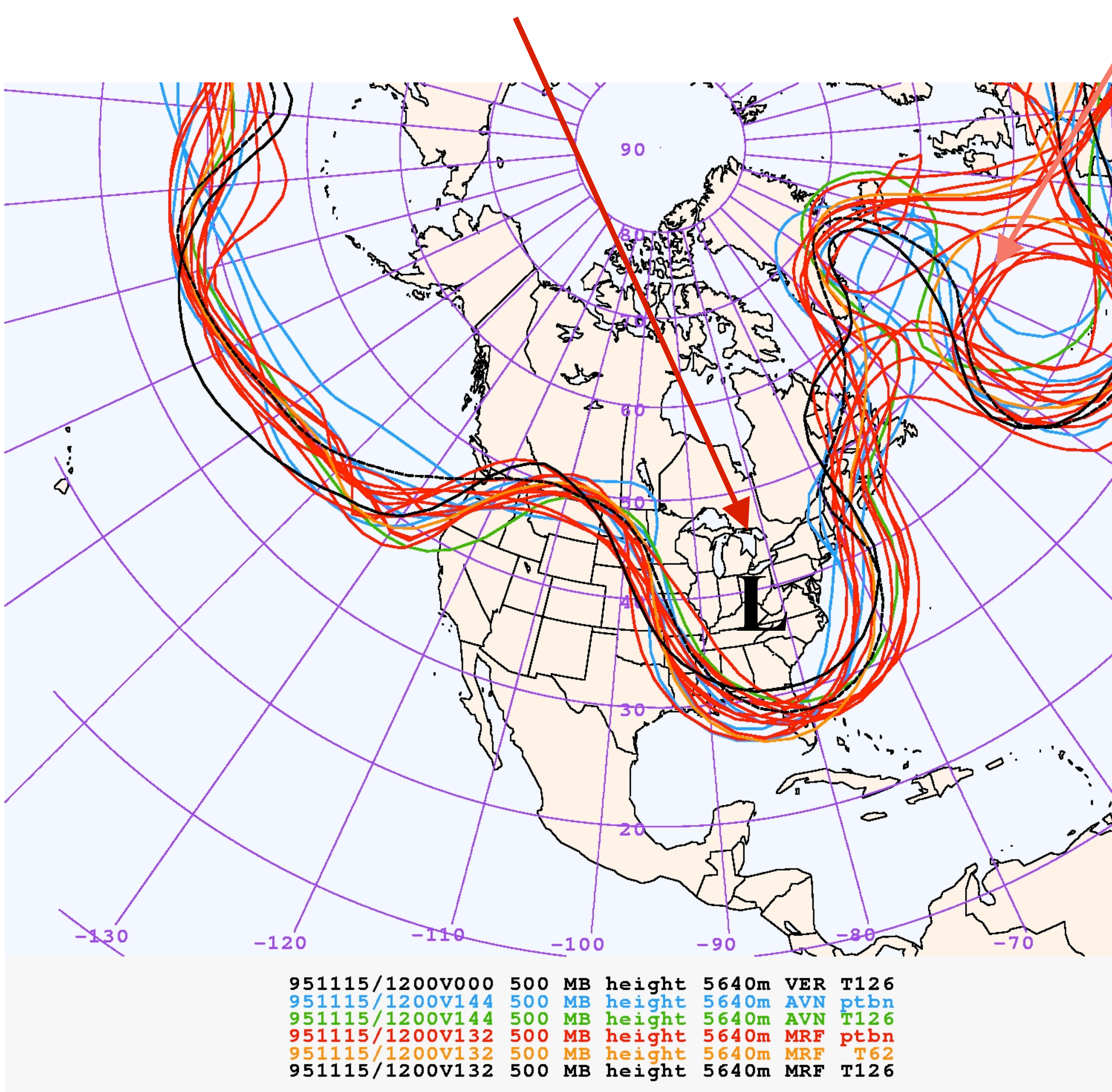
1. Regime change:The presence of red stars (fast BV growth) indicates that the next orbit will be the last one in the present regime.
2. Regime duration: One or two red stars, next regime will be short. Several red stars: the next regime will be long lasting.

These rules surprised Lorenz himself!

These are very robust rules, with skill scores > 95%

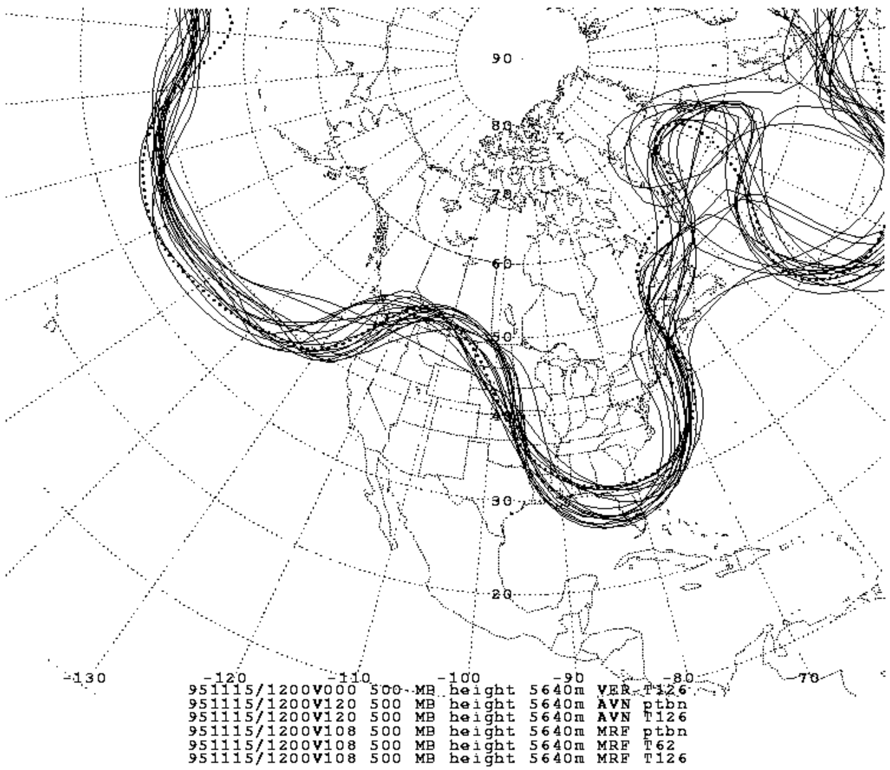


Example of a very predictable 6-day forecast, with "errors of the day"



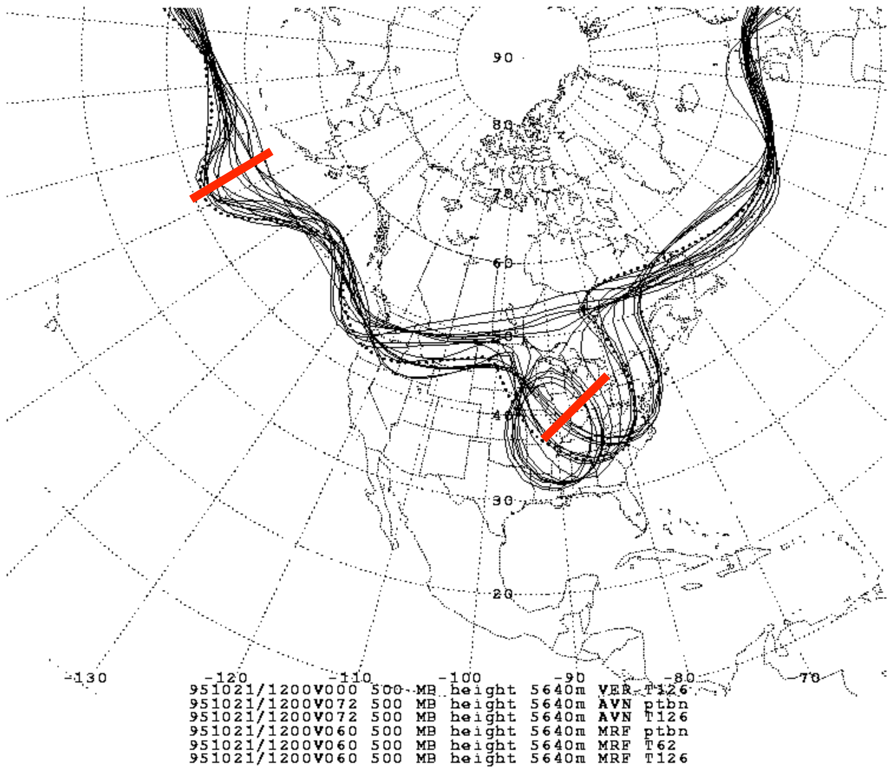
The bred vectors are the growing atmospheric perturbations: "errors of the day"

The errors of the day are instabilities of the background flow. At the same verification time, the forecast uncertainties have *the same shape*



4-day forecast verifying on the same day

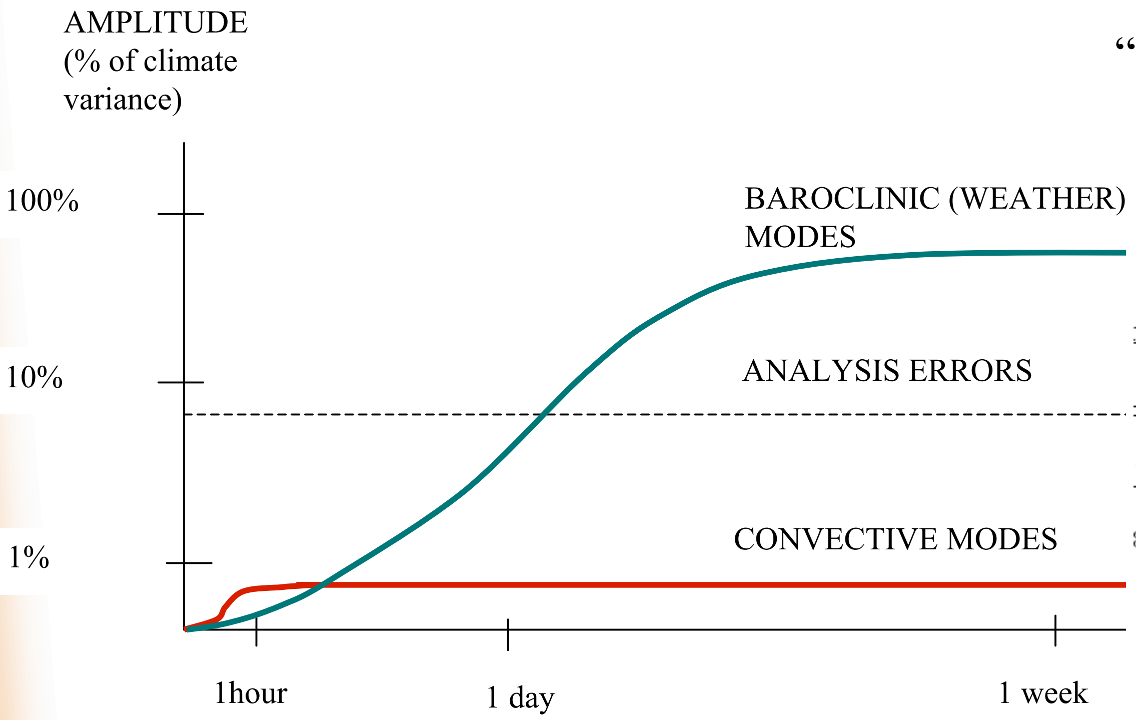
Strong instabilities of the background tend to have simple shapes (perturbations lie in a low-dimensional subspace of bred vectors)



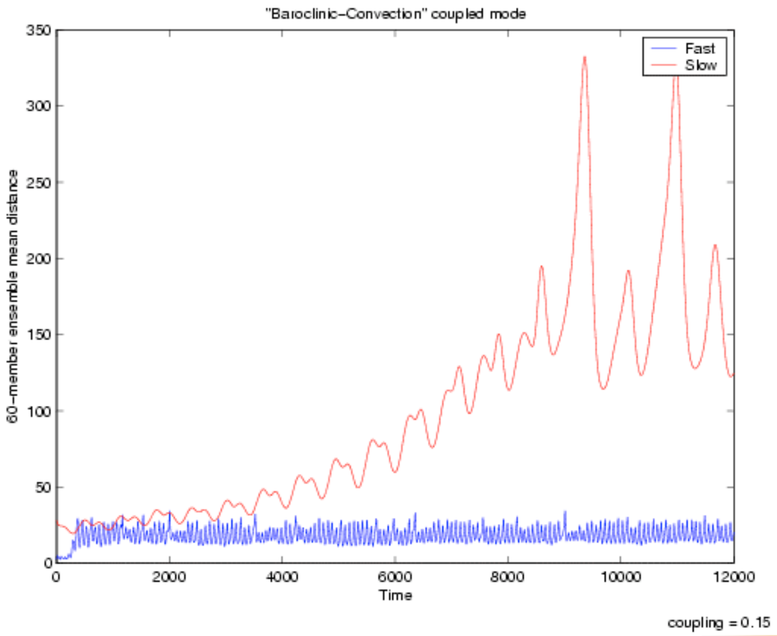
2.5 day forecast verifying on 95/10/21.

Note that the bred vectors (difference between the forecasts) lie on a 1-D space

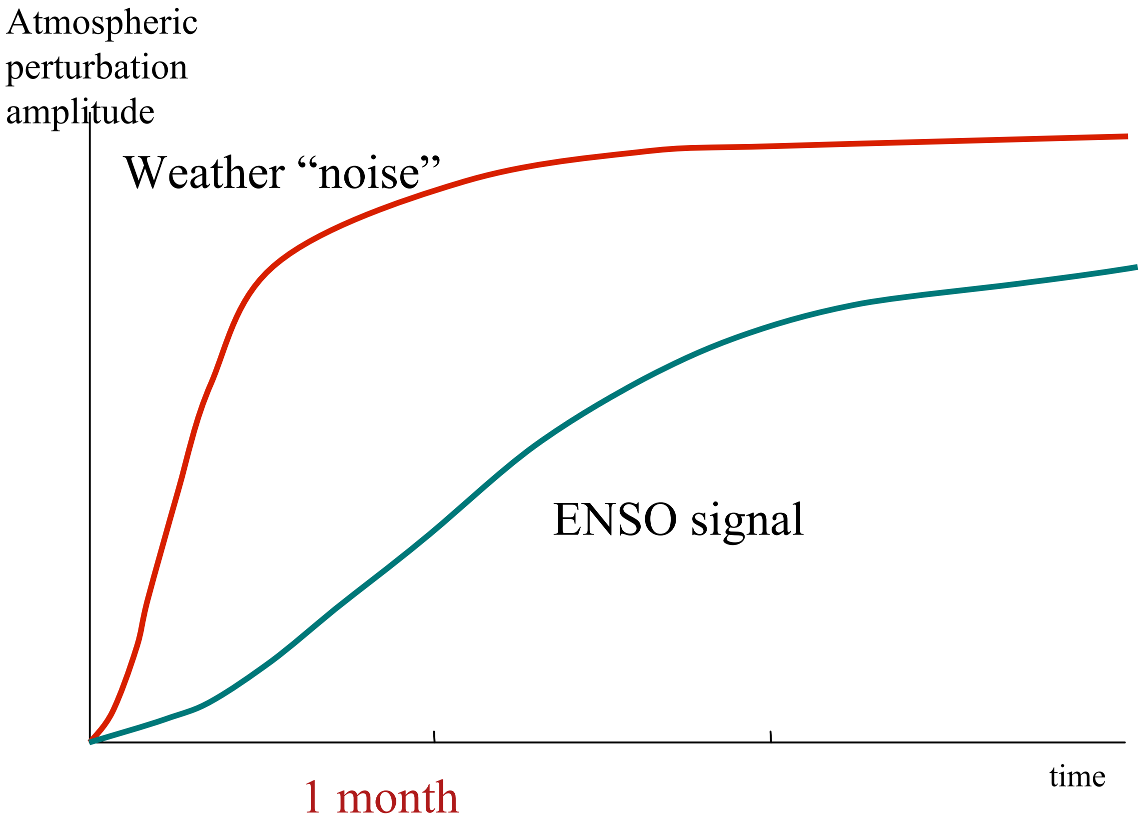
Nonlinear saturation allows filtering unwanted fast, small amplitude, growing instabilities like convection (Toth & Kalnay, 1993, Peña & Kalnay, 2003, NPG)



“weather + convection” coupled model



In the case of coupled ocean-atmosphere modes, we cannot take advantage of the small amplitude of the “weather noise”! Must use the fact that the coupled ocean modes are slower…



Need a long rescaling interval, like 2 weeks or one month

Breeding in a coupled system

- Breeding: finite-amplitude, finite-time instabilities of the system (~Lyapunov vectors)
- In a coupled system there are fast and slow modes, and a **linear** Lyapunov approach (like Singular Vectors) will only capture fast modes.
- Can we do breeding of the slow modes?

We coupled slow and a fast Lorenz (1963) 3-variable models (Peña and Kalnay, 2004)

Fast equations

$$\frac{dx_1}{dt} = \sigma(y_1 - x_1) - C_1(Sx_2 + O)$$

$$\frac{dy_1}{dt} = rx_1 - y_1 - x_1 z_1 + C_1(Sy_2 + O)$$

$$\frac{dz_1}{dt} = x_1 y_1 - bz_1 + C_1(Sz_2)$$

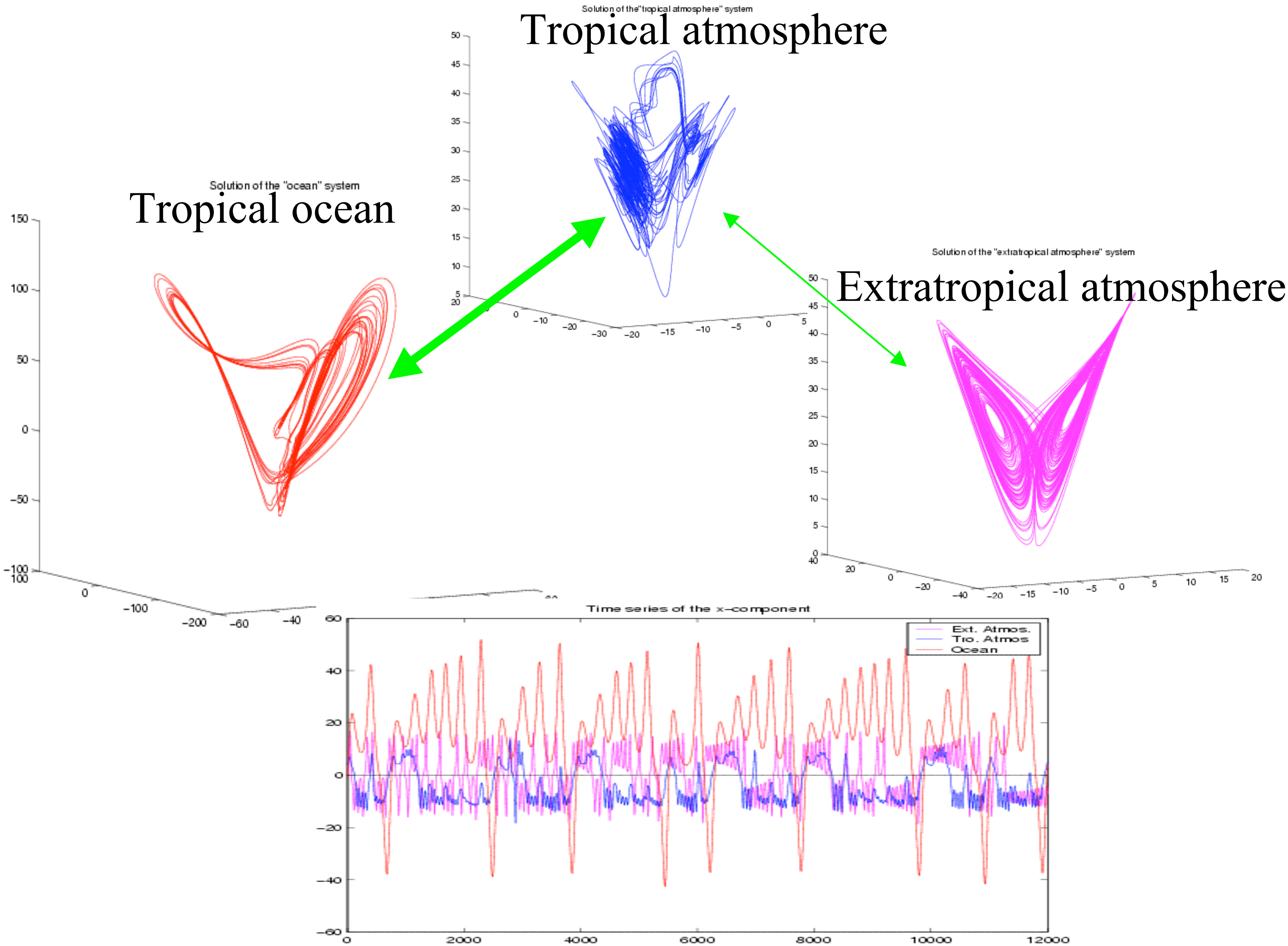
Slow equations

$$\frac{1}{\tau}\frac{dx_2}{dt} = \sigma(y_2 - x_2) - C_2(x_1 + O)$$

$$\frac{1}{\tau}\frac{dy_2}{dt} = rx_2 - y_2 - Sx_2 z_2 + C_2(y_1 + O)$$

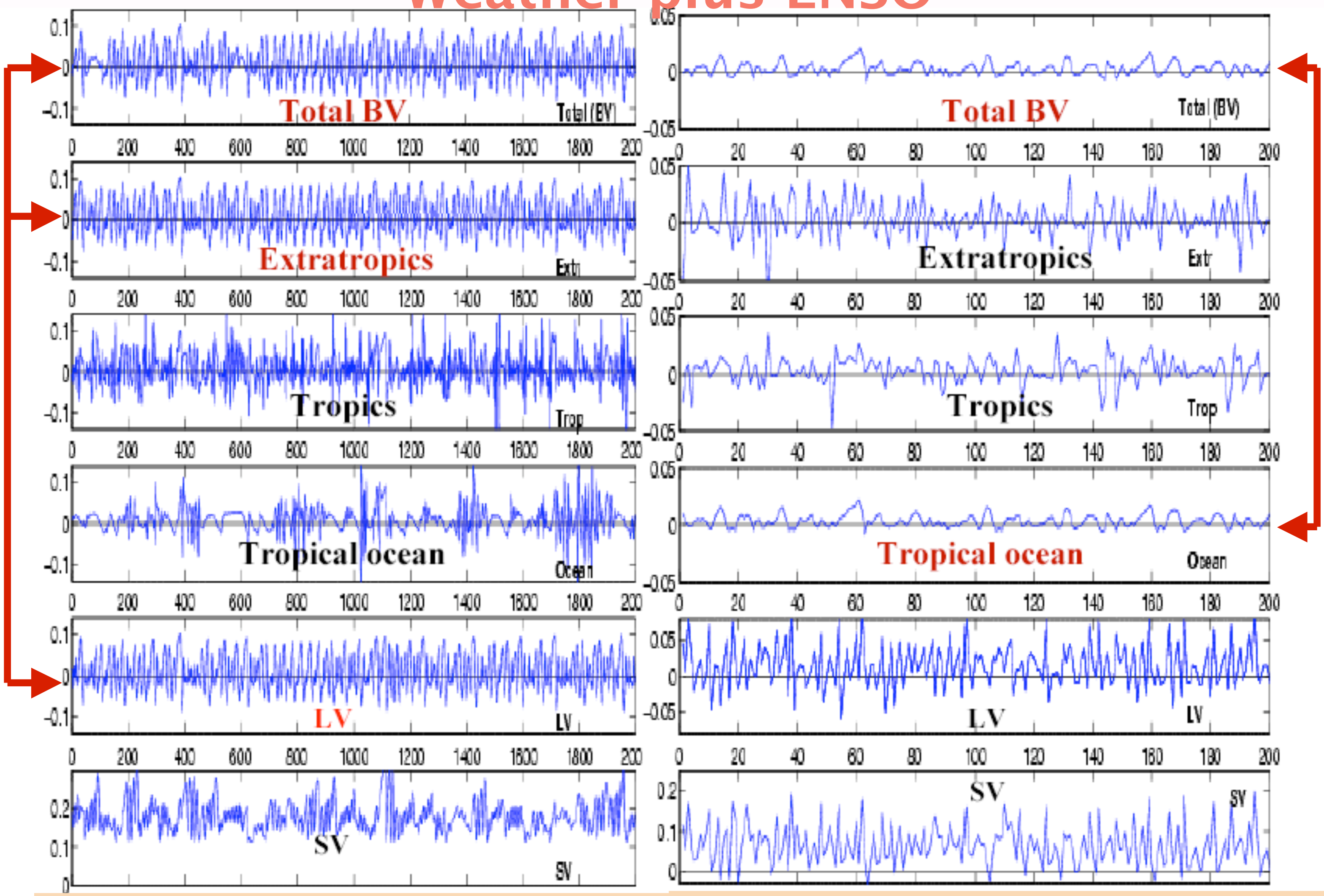
$$\frac{1}{\tau}\frac{dz_2}{dt} = Sx_2 y_2 - bz_2 + C_2(z_1)$$

"Tropical-extratropical" (triply-coupled) system: the ENSO tropical atmosphere is weakly coupled to a fast "extratropical atmosphere" with weather noise



Solution of the"tropical atmosphere" system
Tropical atmosphere
Solution of the "ocean" system
Tropical ocean
Solution of the "extratropical atmosphere" system
Extratropical atmosphere
Time series of the x-component
Ext. Atmos.
Tro. Atmos
Ocean

Breeding in a coupled Lorenz model: "Weather plus ENSO"



Short rescaling interval (5 steps) and small amplitude: fast modes

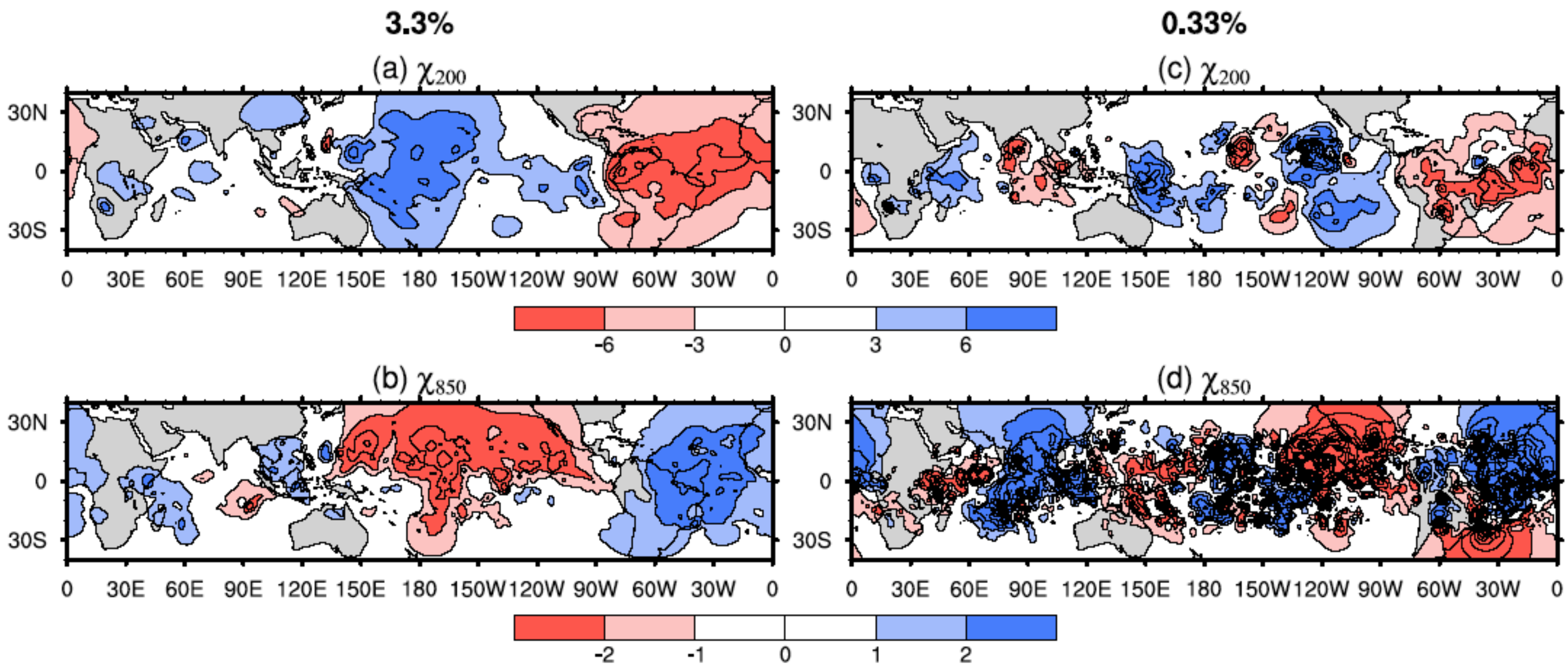
Long rescaling interval (50 steps) and large amplitude: ENSO modes

The linear approaches (LV, SV) cannot capture the slow ENSO signal

From Lorenz coupled models:

- In coupled fast/slow models, we can do breeding to isolate the slow modes
- We have to choose a slow variable and a long interval for the rescaling
- This is true for nonlinear approaches (e.g., EnKF) but not for linear approaches (e.g., SVs, LVs)
- We apply this to ENSO coupled instabilities:
  - Cane-Zebiak model (Cai et al, 2003 JC)
  - NASA/NCEP fully coupled GCM (Yang et al, 2006 JC)
  - NASA operational system with real observations (Yang et al 2007, MWR)

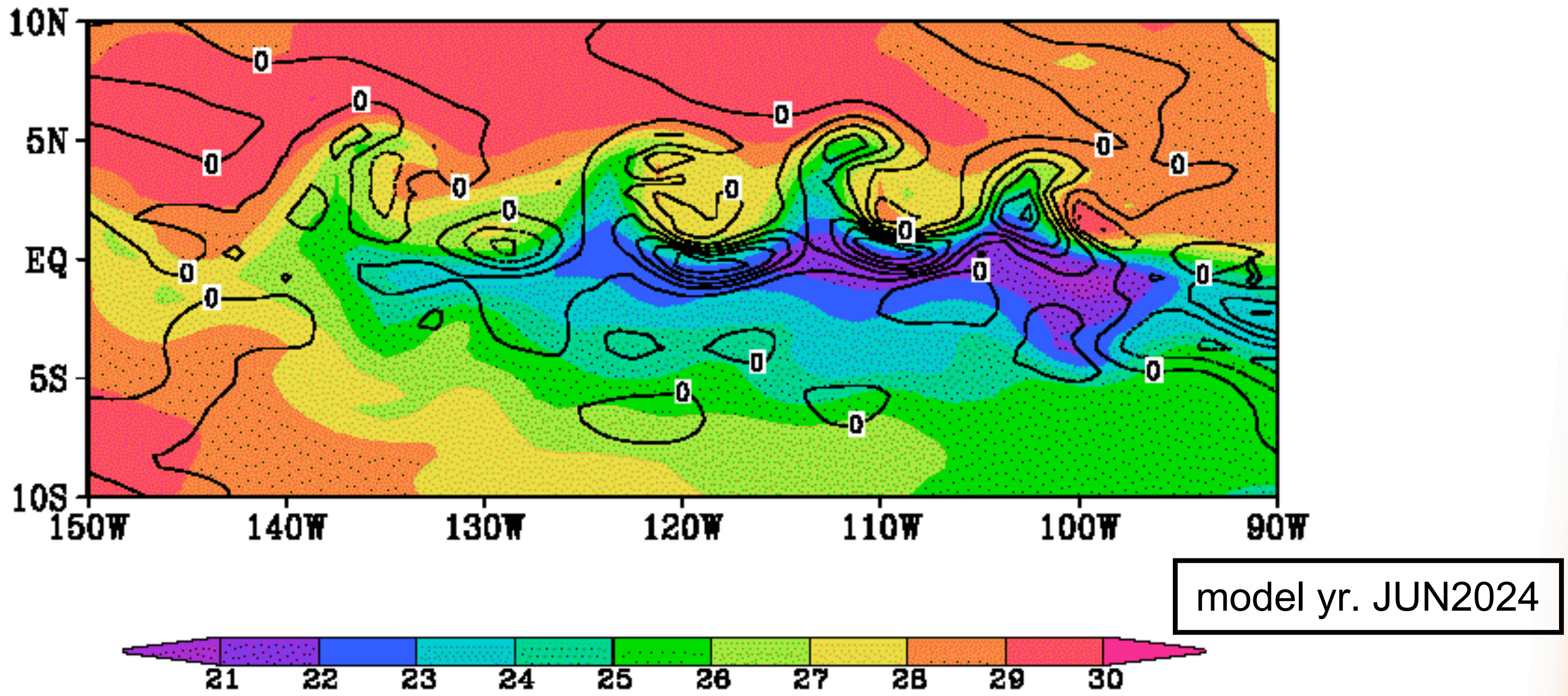
Chikamoto et al (2007, GRL): They found the Madden-Julian instabilities by choosing appropriately the rescaling amplitude (only within the tropics)



Breeding method

- **Bred vectors** :
  The differences between the control forecast and perturbed runs
- **Tuning parameters**
  - Size of perturbation (e.g., Niño–3 SST)
  - Rescaling period: one month
- **Advantages**
  - Low computational cost
  - Easy to apply to Coupled GCMs
  - Captures coupled instabilities

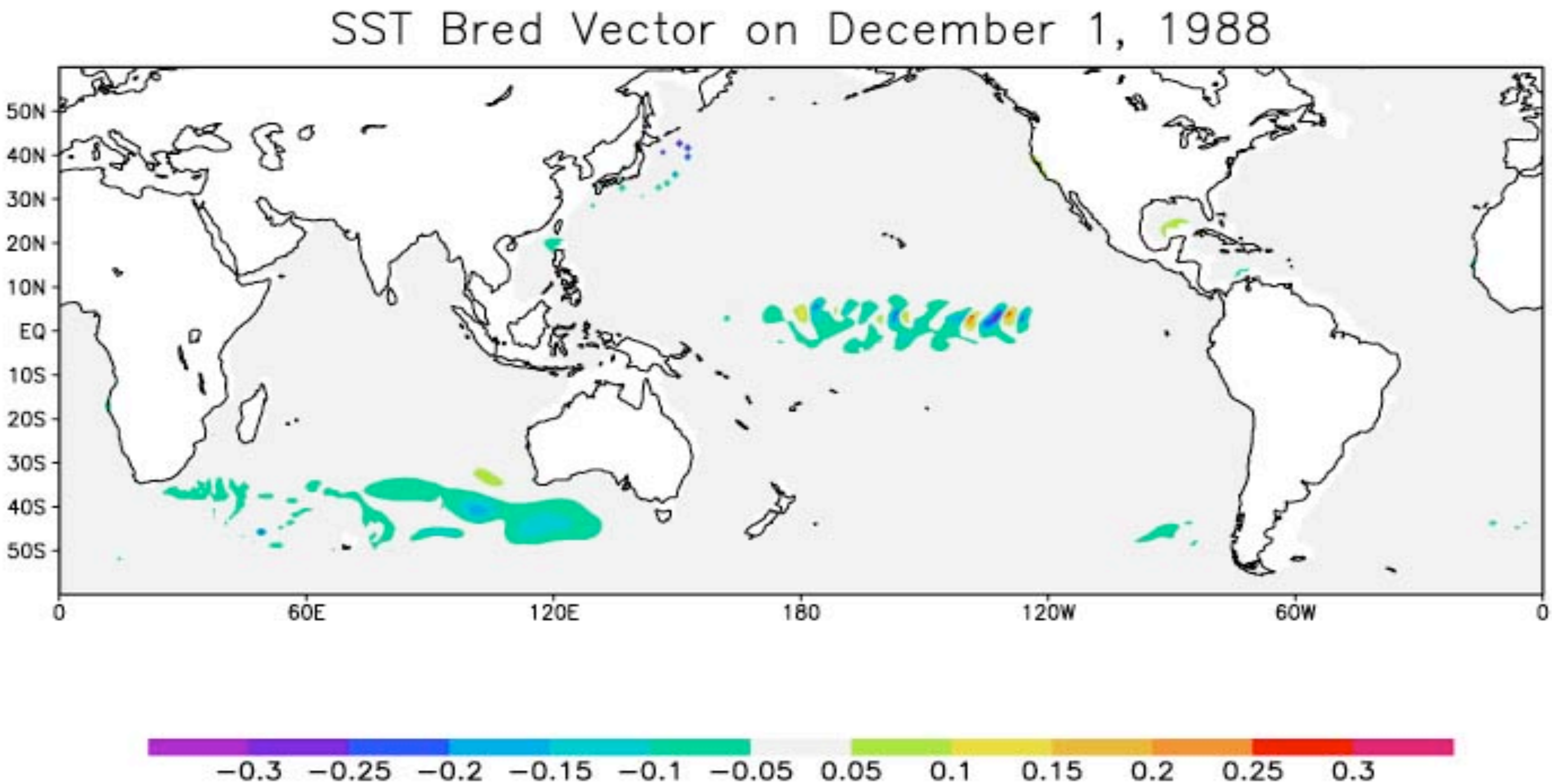
Yang et al 2006: Example of instantaneous background SST (color) and bred vector SST (contours)



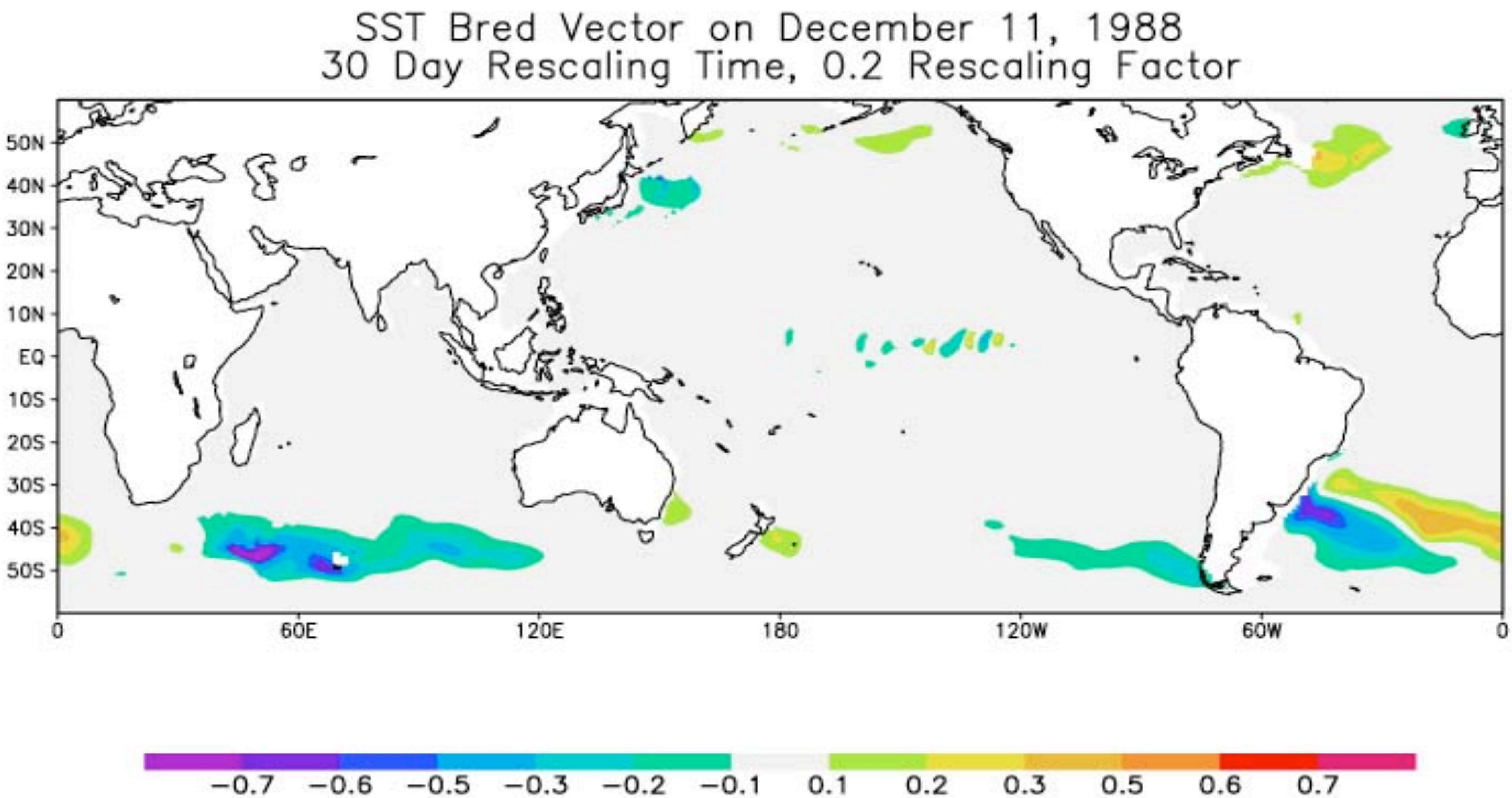
Instabilities associated with the equatorial waves in the NSIPP coupled model are naturally captured by breeding!

Hoffman et al (2007): finding all ocean instabilities with breeding time-scale 10-days captures tropical instabilites

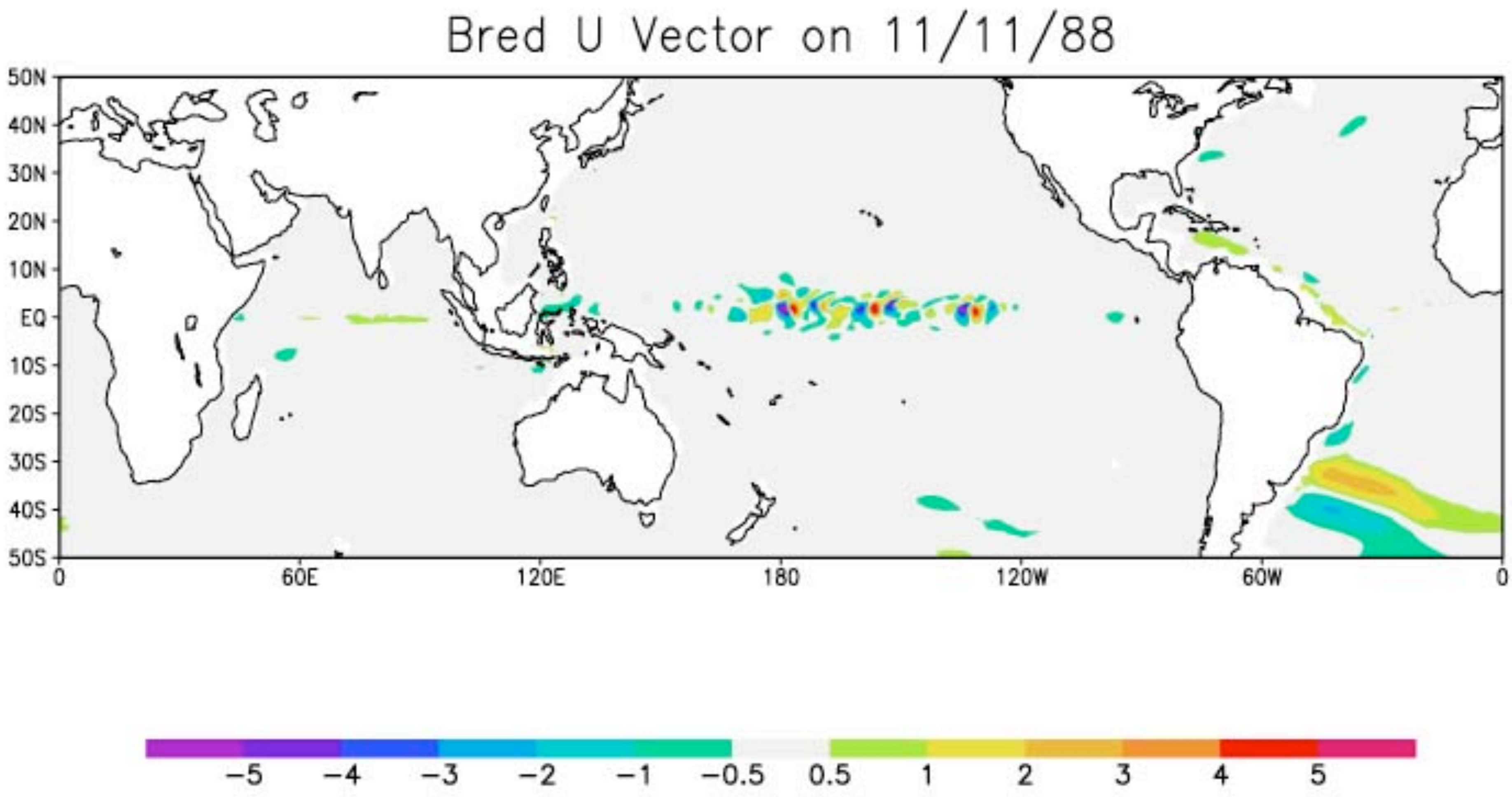
Breeding time scale: 10 days



When the rescaling time scale is 30 days, extratropical instabilities dominate



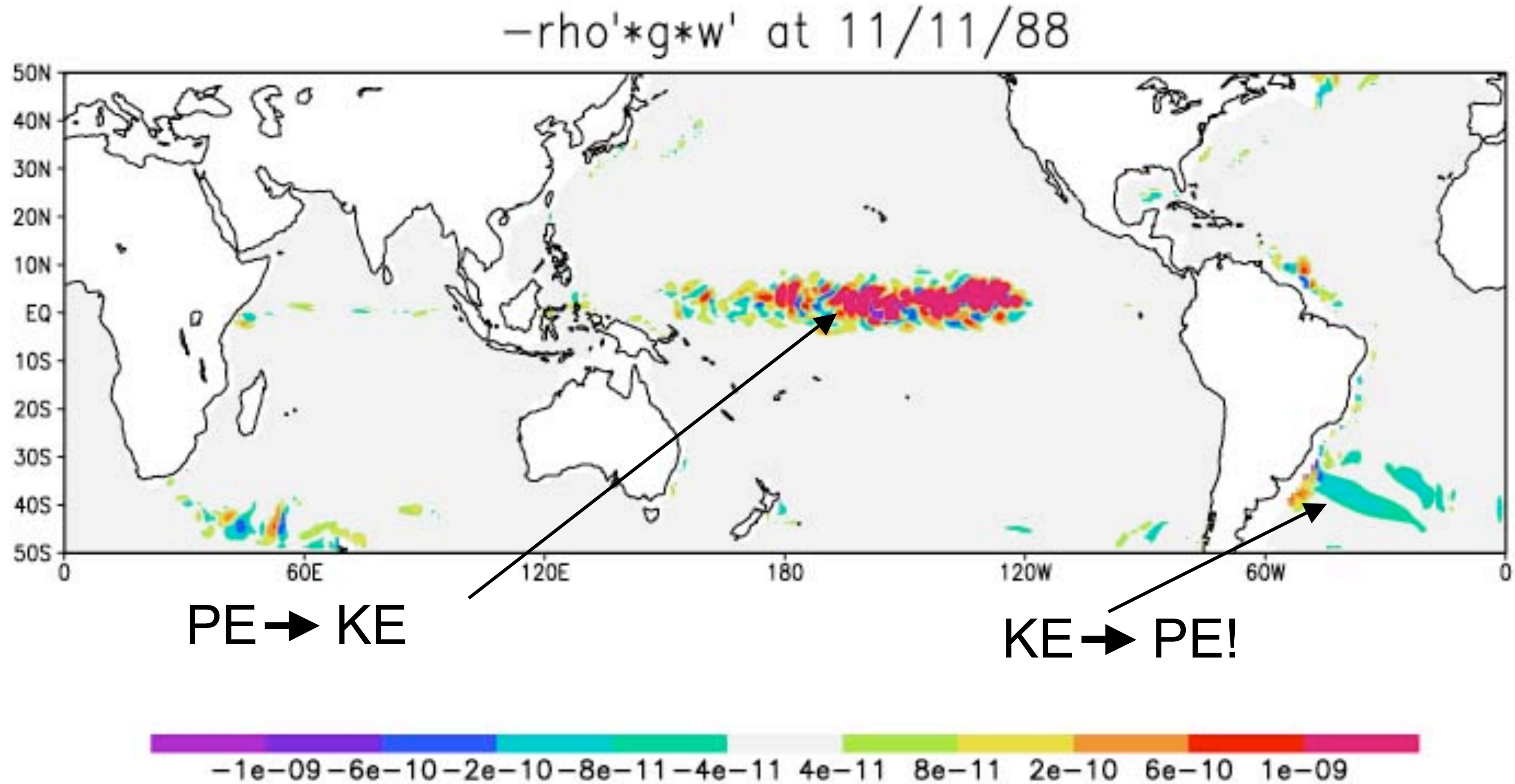
Here we have both tropical and "South Atlantic Convergence Zone" instabilities. Can we determine the dynamic origin of the instabilities?



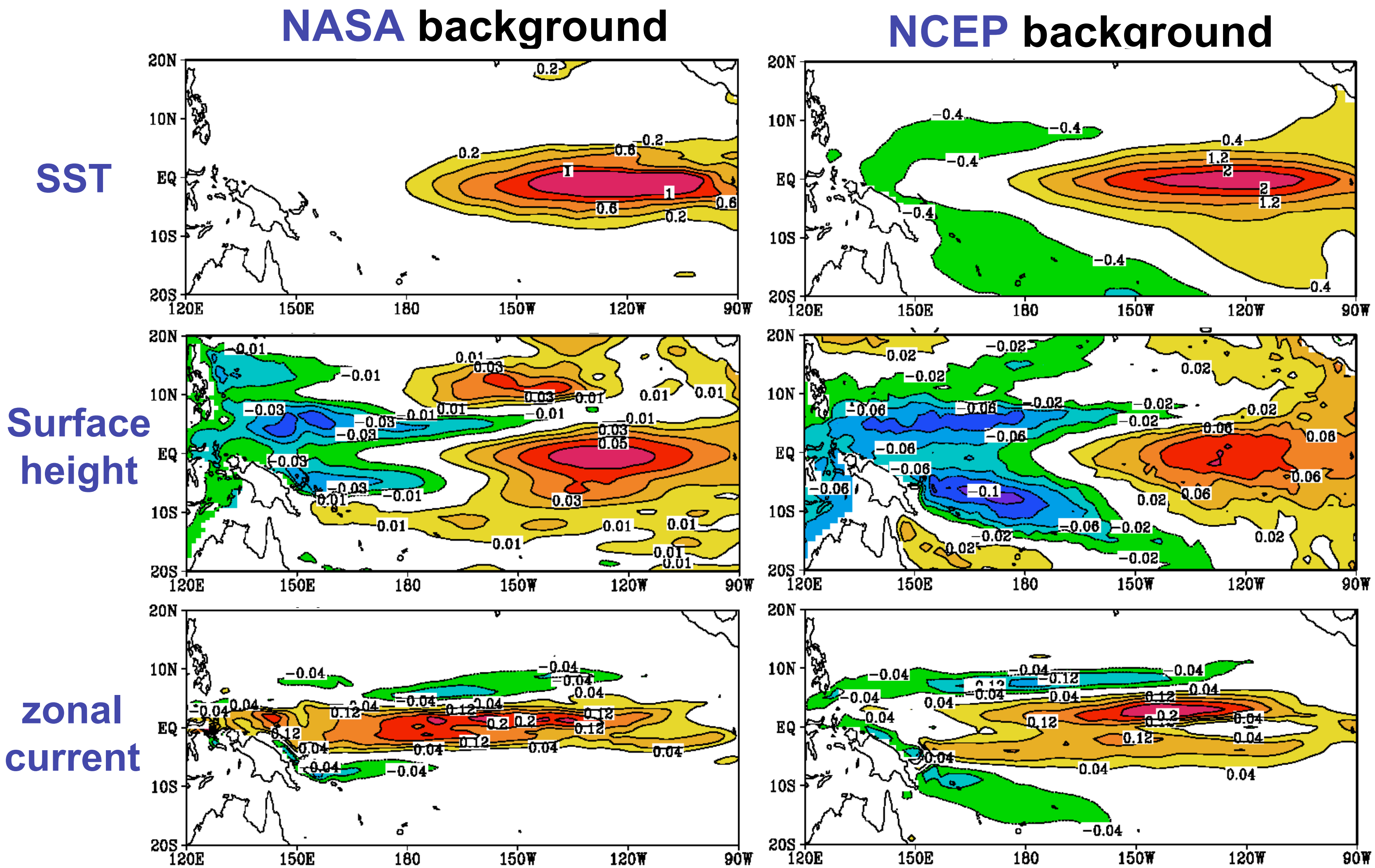
With the Bred Vector kinetic energy equation it is easy… (both control and perturbed V satisfy the full equations)! Example: Baroclinic conversion term

$$\frac{\partial e_b}{\partial t} = -\nabla \cdot \left(\vec{V}_c e_b\right) - \nabla \cdot \left(\vec{V}_b p_b\right) \underline{- w_b \rho_b g}$$

$$-\rho_0 \left[\vec{V}_b \cdot \left(\vec{V}_b \cdot \nabla\right)\vec{V}_c + \vec{V}_b \cdot \left(\vec{V}_b \cdot \nabla\right)\vec{V}_b - \vec{V}_b \cdot \left(\vec{V}_c \cdot \nabla\right)\vec{V}_b\right] - \frac{\partial}{\partial z}\left(w_c e_b\right) - \frac{\partial}{\partial z}\left(w_b \vec{V}_c\right) + \frac{\partial}{\partial z}\left(w_b p_b\right) + \vec{F}_b$$

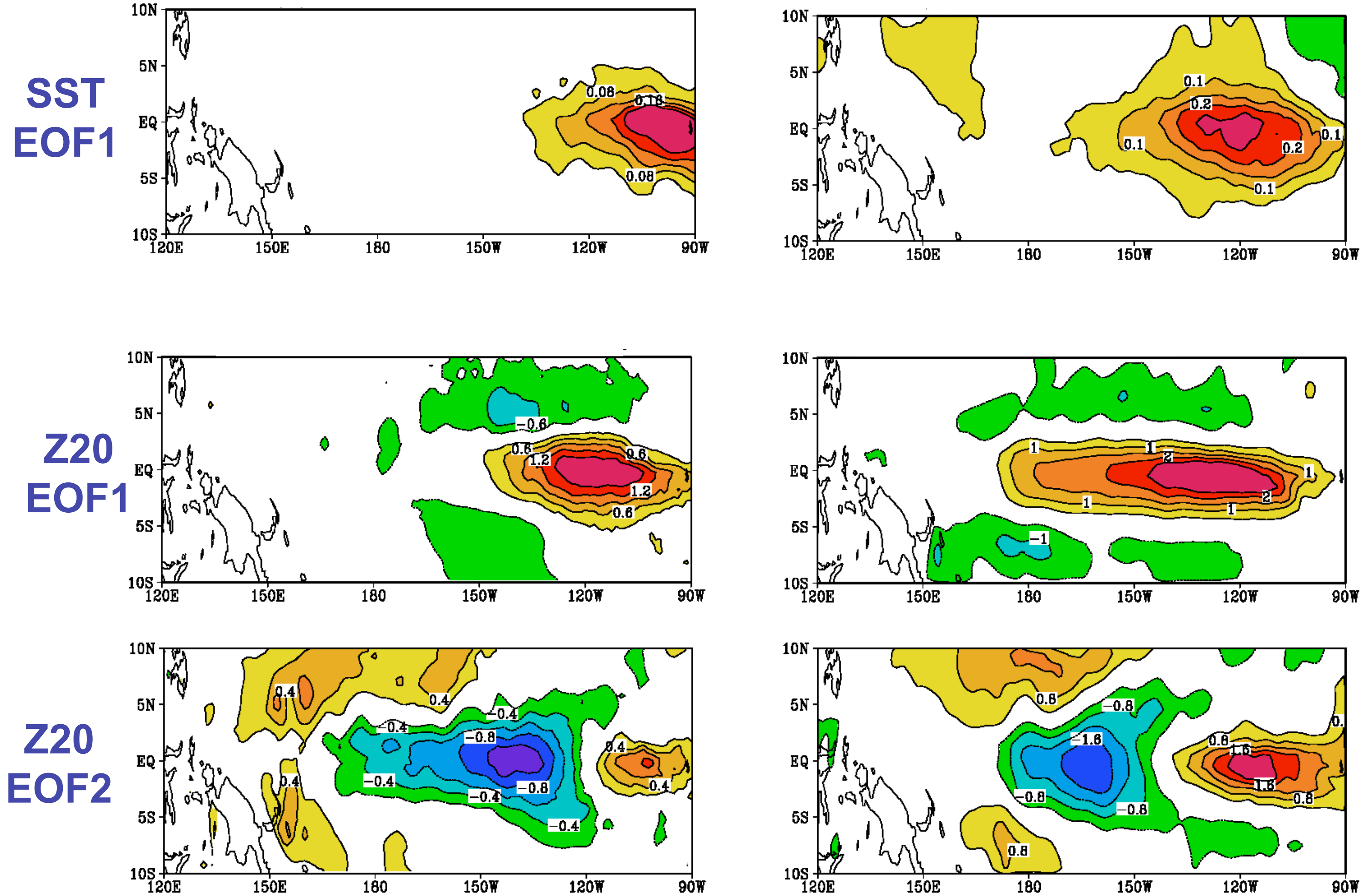


Yang 2006: NASA and NCEP Coupled GCMs (ENSO mode regressed with Niño-3 SST)



These two coupled models have slightly different ENSOs…

NASA Bred Vector vs. NCEP Bred Vector

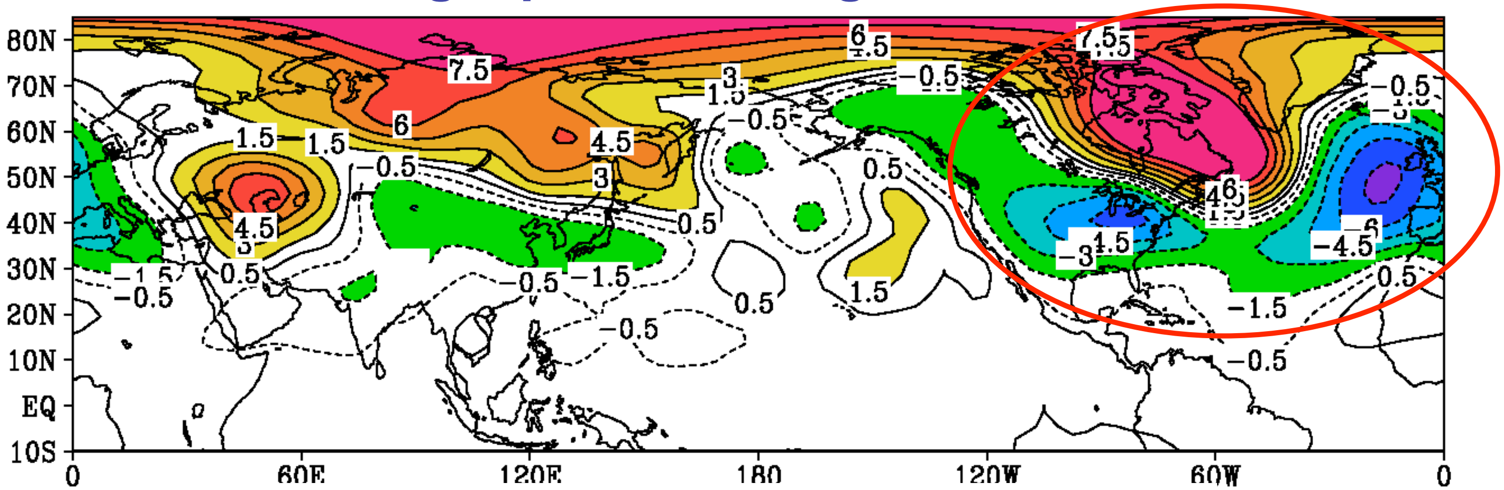


Bred vectors obtained with an 8-year NCEP run are extremely similar to the NASA's 20-year run!!!

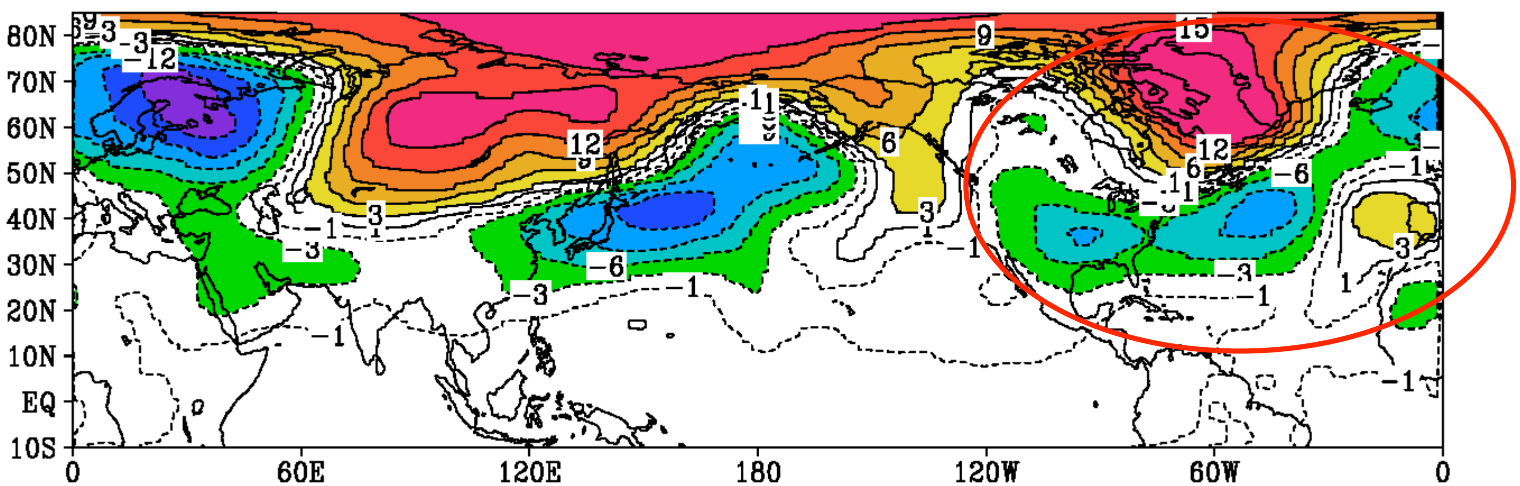
NASA BV vs. NCEP BV

Northern Hemisphere

NASA geopotential height at 500mb



NCEP geopotential height at 500mb



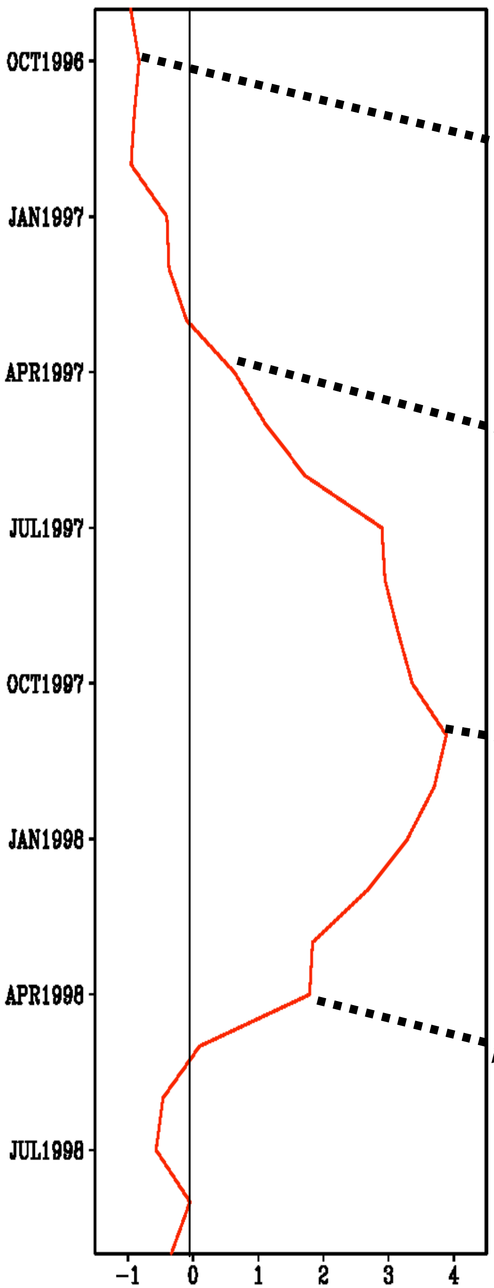
Even the PNA atmospheric teleconnections are similar!!

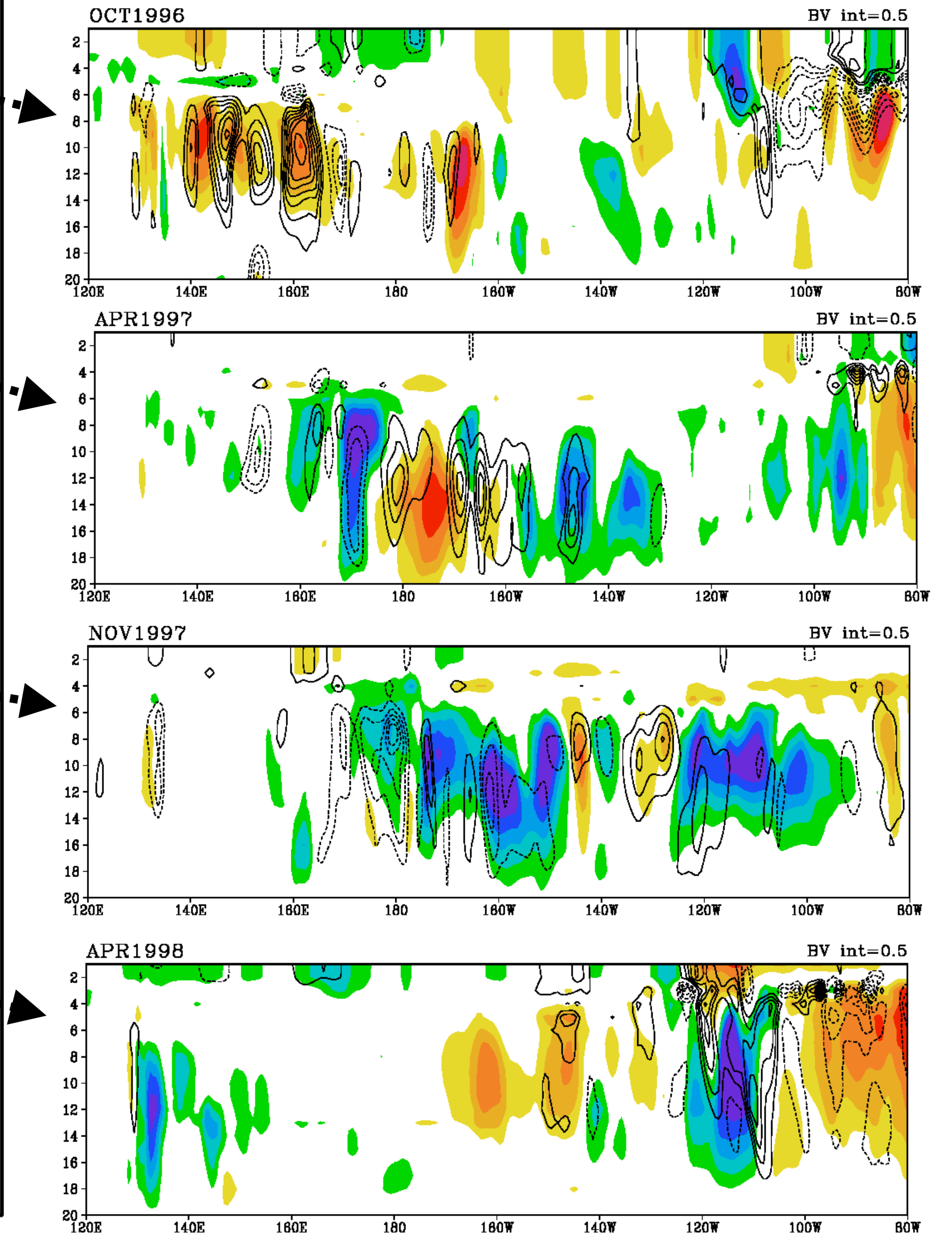
Yang: NASA Seasonal-to Interannual Prediction (NSIPP) coupled GCM

Components	•AGCM ◆Developed by Suarez (1996) ◆Resolution: 2°× 2.5°×34 levels •OGCM/Poseidon V4 ◆Developed by Schopf and Loughe (1995) ◆Resolution: 1/3°× 5/8°× 27 layers •Mosaic LSM

- Full globally coupled model
- AGCM and OGCM coupled everyday
- Current prediction skill (El Niño hindcasts) is up to 9 months

Yang (2005) Vertical cross-section at Equator for BV (contour) and 1 month forecast error (color)





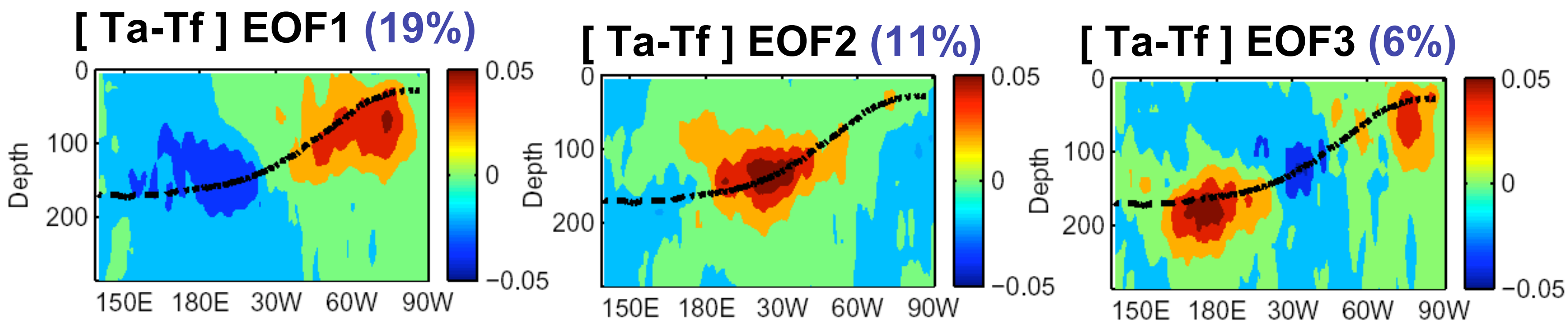
Before 97' El Niño, error is located in W. Pacific and near coast region

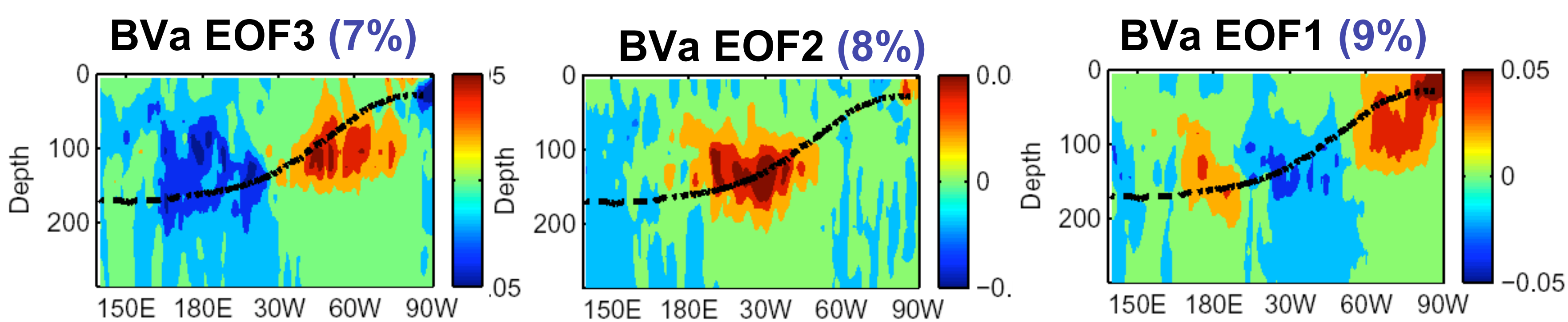
During development, error shifts to lower levels of C. Pacific.

At mature stage, error shifts further east and it is smallest near the coast.

After the event, error is located mostly in E. Pacific.

The equatorial temperature 1-month forecast error structure

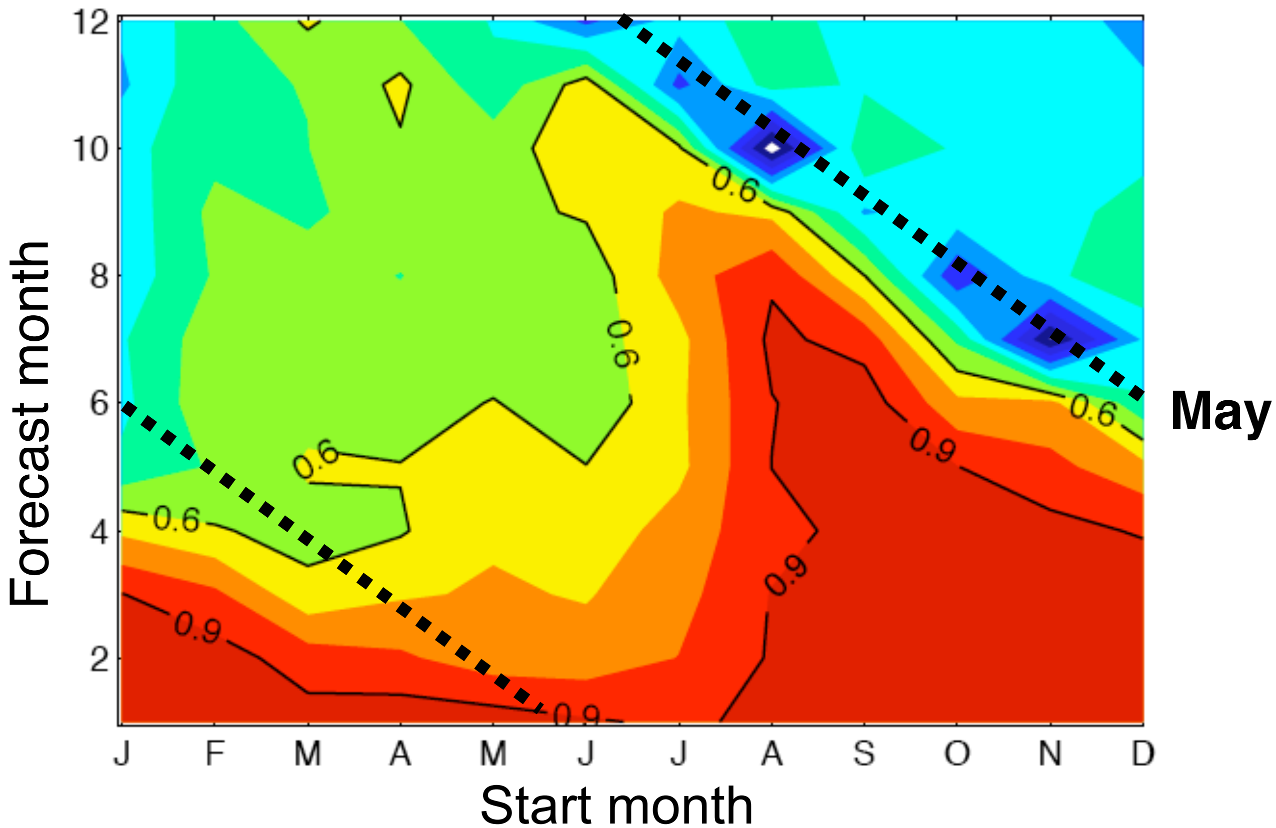




The equatorial BV structure

BV and forecast errors have very similar subsurface thermal structure!

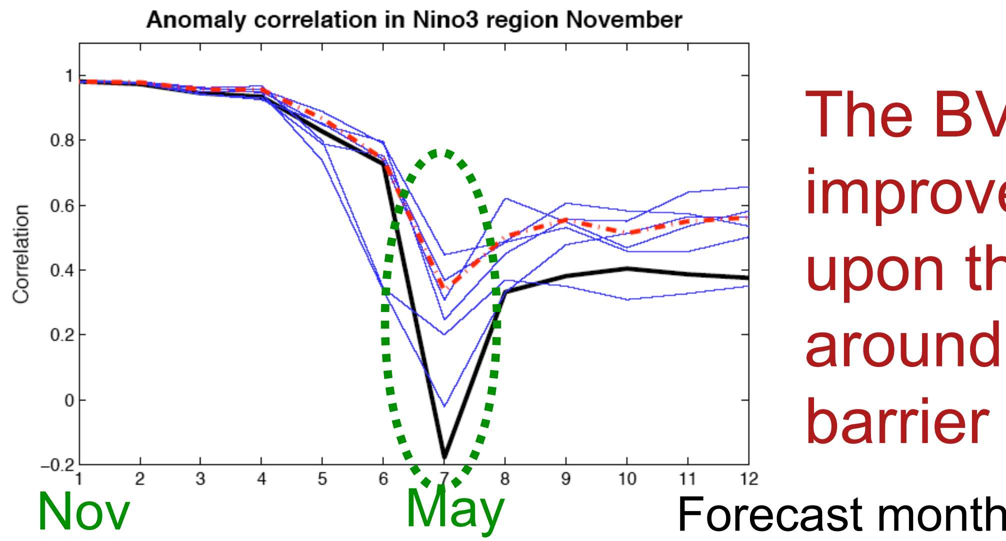
Anomaly correlation between control hindcast and Reynolds SST in Niño3 region



- Starting from the cold season, forecast skill is high for the first 6 months, but they have difficulties overcoming the “spring barrier”.
- When starting from the warm season, forecast skill quickly drops.

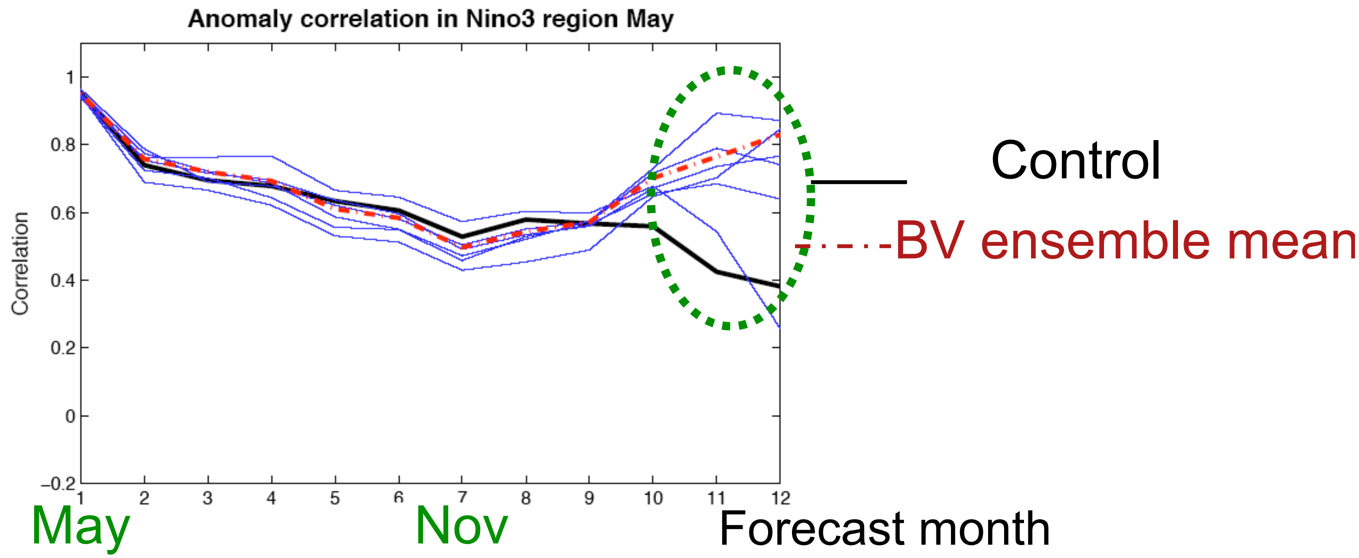
Anomaly correlation with 3 pairs of BVs November and May restarts (1993-2002)

Start from
cold season



The BV ensemble improves a lot upon the control around the Spring barrier in skill!

Start from
warm season



Summary: breeding is easy and powerful

- Nonlinear methods, like breeding and EnKF, can take advantage of the saturation of fast weather noise. Linear systems cannot.
- Coupled Lorenz model experiments show that for slow modes the rescaling in breeding has to be done using slow variables and long rescaling intervals
- Cane-Zebiak breeding experiments show that the BV growth depends on season and ENSO phase, and that they can be used for data assimilation and ensemble forecasting
- "Perfect model" experiments with the NASA and NCEP coupled GCMs show a robust dominant coupled ocean/atmosphere bred vector.
- Results generally agree with those obtained with the C-Z model
- BV can easily explain the physical origin of ocean instabilities
- Bred Vectors predict well the evolution of forecast errors
- Ensembles of BV improve the seasonal and interannual forecast skill, especially during the "spring barrier"

Current Applications

- Bred vectors will be implemented as initial coupled perturbations for ensemble ENSO forecasting in the NASA NSIPP operational system.
- Because they can detect the month-to-month background error variability, bred vectors are being tested to improve **oceanic data assimilation**.
- We will explore all ocean instabilities and their origin

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